Homework

December 16, 2019

1 Lecture 9

1. Consider function

$$f_\mu(x) = \max_{u \in Q_u} \{\langle Ax, u \rangle_u - h(u) - \mu d_u(u)\},$$

where $d_u(u)$ is 1-strongly convex distance generating function, $h(u)$ is convex. Prove that the gradient $\nabla f_\mu(x) = A^* u_\mu(x)$ is $\frac{\|A\|^2_{x\to u}}{\mu}$-Lipschitz-continuous. Here

$$u_\mu(x) = \arg\max_{u \in Q_u} \{\langle Ax, u \rangle_u - h(u) - \mu d_u(u)\}.$$

2. Obtain a smoothed counterpart of the function

$$f(x) = \max_{j=1,\ldots,m} |\langle a_j, x \rangle_x - b_j|.$$

3. Propose a universal modification of Dual Gradient Method.
4. Consider a problem

$$\min_x \{f(x) = \sum_{i=1}^{m} \alpha_i |x_i|^3 : Ax = b\}.$$

Write the dual problem and show that its objective has Hölder-continuous subgradient.